Convex Storage Loss Modeling for Optimal Energy Management

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Journées Nationales Automatique 2020 (JNA),
Session Commande des Systèmes Électriques (CSE)
November 26, 2020

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Outline of the presentation

1. Context
2. Convex storage modeling
   - Generic storage model
   - Limitation of the linear loss model
   - Relaxation of storage losses
3. Panorama of storage loss models
   - Overview of loss models
   - Existing models
   - Our contribution
4. Loss models illustrated
5. Conclusion

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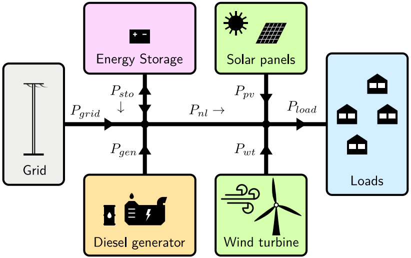
Energy management (EM) is often optimization based

EM = control of the power flows in a system with *storages*



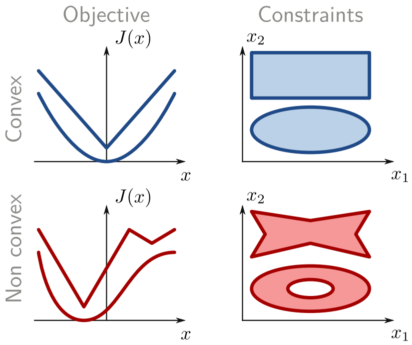
Control objective:

- Minimizing a criterion (economical, ecological…)
- Satisfaying constraints (e.g. storage bounds)

EM is often based on *online* optimization (e.g. MPC)
→ optimization should be *100% reliable* → **convex**

Convex optimization problems can be solved reliably

$$\min_{x \in \mathbb{R}^n} J(x), \text{ s.t. } g(x) \leq 0, \; h(x) = 0$$



Conditions this optimization to be convex

Convex objective J, convex ineq. func. g, **linear** eq. func. h

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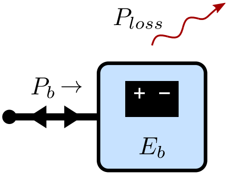
Generic storage model

Storage dynamics with losses is *linear:*

$$E_b(k+1) = E_b(k) + (P_b(k) - P_{loss})\Delta_t$$

→ Convexity of the storage model depends on the *convexity of the loss expression*



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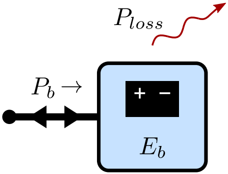

→ Convexity of the storage model depends on the *convexity of the loss expression*

Now, what choice of loss expression: $P_{loss} = ?$

- Convex for efficient optimization
- and also physically realistic?

Linear loss model: very limiting

Linear expression is the only *genuine* convex model:

$$P_{loss} = p_0 + c_P P_b + c_E E_b$$

Lossless storage (P_{loss}) is the popular special case

Coefficient meaning:

- p_0: constant self-discharge
- c_E: self-discharge proportional to energy level
- c_P: physically *not meaningful*

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Limitation

Unfortunately, many simple expressions like $P_{loss} = P_b^2$ are *not convex*

Relaxation of storage losses

Key idea (widely used in literature)

Relax the equality constraint (losses = some expression) to an inequality (losses $\geq$ some expression).

$\longrightarrow$ This allows using *any convex expression* for losses

Generic loss formulation (assuming a dependency on storage power and energy):

$$P_{loss} \geq g(P_b, E_b)$$

where g means any *convex* function.

Exactness of the relaxation of losses

Although losses relaxed as $P_{loss} \geq g(P_b, E_b)$, it is expected that, at the optimum, the inequality will be *tight*.

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Heuristic justification: "Positive price argument"

If the incremental cost of wasting energy is *positive*, then at the optimum, no energy should be wasted.

Otherwise (negative energy price), there can be an *artificial excess* of wasted energy (excess: $P_{loss} - g > 0$).

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A few sets of sufficient conditions were reported, but:

- application specific
- cannot always be checked *ex-ante*
- conditions are sufficient, but not necessary

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Possibilities for loss expressions

Existing convex loss expressions (beyond linear):

- Piecewise-linear-in-P (i.e. **constant efficiency**):
  Physics-free, widespread usage
- Quadratic-in-P:
  approx. Joule heating ($r.I^2$)
- Quadratic-in-P over linear-in-E (P^2/E):
  approx. Joule heating in a capacitor

Our proposition: the "convex monomial loss model" ($\sim P^a/E^b$)

One continous family of nonlinear convex loss model

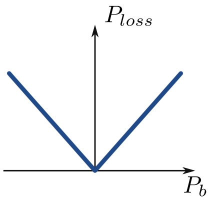
- parametrized by 4 (or 8) real coefficients:
  suitable for *model fitting* to experimental loss data
- contains all existing models as special cases

Piecewise-linear-in-P loss model

Loss expression:

$$g(P_b, E_b) = c_+ P_b^+ + c_- P_b^-$$

with P_b^+ and P_b^-: positive and neg. parts of P_b
Property: physics-free



This loss model corresponds to the ubiquitous **constant efficiency** storage model*:

$$E_b(k+1) \leq E_b(k) + (\eta_+ P_b^+ + P_b^- / \eta_-)\Delta_t$$

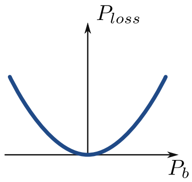
with $\eta_+ = 1 - c_+$ and $\eta_- = 1/(1 + c_-)$

(*) this model is normally written with "=", but it is still a relaxation, because variables P_b^+, P_b^- are not exclusive unless imposed (MILP)

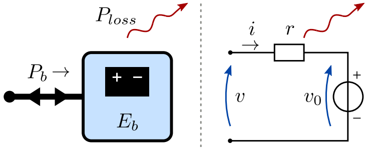
Quadratic-in-P loss model

Loss expression:

$$g(P_b, E_b) = \rho.P_b^2$$



Property: inspired by Joule heating



Correspondence to Joule heating in a circuit: $\rho = r/v_0^2$

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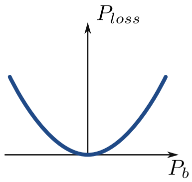

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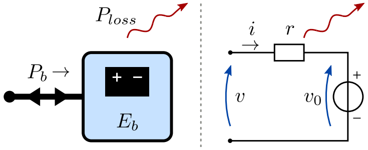

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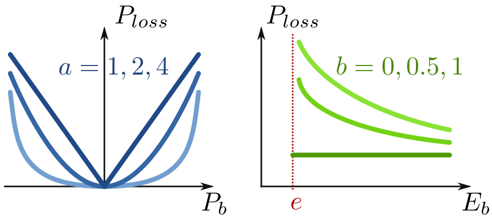
Idea to generalize the quadratic model

Because v_0 and r typically depend on the energy level (*SoE*), search for a *separable* loss model with $\rho(E_b)$

Contribution: the "Convex monomial loss model"

Loss expression (symmetric charge/discharge case):

$$g(P_b, E_b) = c\frac{|P_b|^a}{|E_b - e|^b}$$



Convex with $a \geq 1$, $b \geq 0$ and $b \leq a - 1$

Examples:

- Quad-in-P ($a = 2$) $\longrightarrow$ $b \in [0, 1]$, e.g. P^2/E (capacitor)
- PWL-in-P ($a = 1$) $\longrightarrow$ $b = 0$ $\longrightarrow$ **No SoE effect allowed!**

Contribution: the “Convex monomial loss model”

If charge/discharge asymmetry is wanted/needed

Loss expression:

$$g(P_b, E_b) = c_+ \frac{(P_b^+)^{a_+}}{|E_b - e_+|^{b_+}} + c_- \frac{(P_b^-)^{a_-}}{|E_b - e_-|^{b_-}}$$

A split between positive and negative parts of the power, like the PWL-in-P model.
(needed to implement the absolute value ($a = 1$) with a Linear Program)

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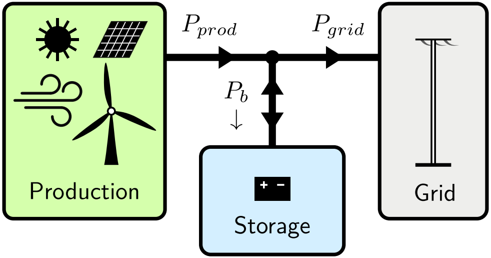
Application: storage for grid-connected production



Optimization objective: energy sold to grid at price c_{grid}

$$\max C_{grid} = \sum_{k=1}^{K} c_{grid}(k).P_{grid}(k)\Delta_t$$

Scenario description: production shifting

Parameters: 2 hours ($K = 20, \Delta_t = 0.1\,\text{h}$), $E_{rated} = 1\,\text{kWh}$ storage

1. 1st hour: prod. $P_{prod} = 1\,\text{kW}$ and *low* price $c_{grid} = 0.1\,€/\text{kWh}$
2. 2nd hour: *zero* production and *high* price $c_{grid} = 0.2\,€/\text{kWh}$

Interpretation

Storage can shift the production to the high price hour

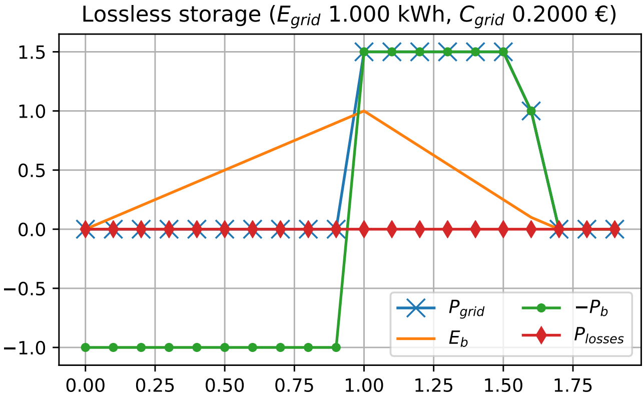
Experiment objective

See the effect of the loss model on the charge/discharge profile

Remark: All loss models calibrated for same 80% round-trip storage efficiency on the experiment

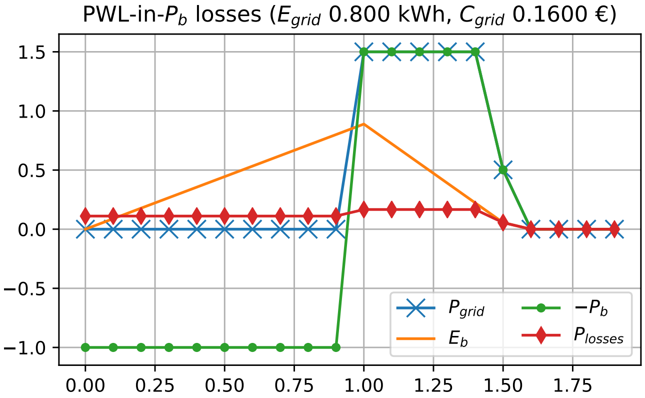
Production shifting experiment

Case 1: Lossless storage



Production shifting experiment

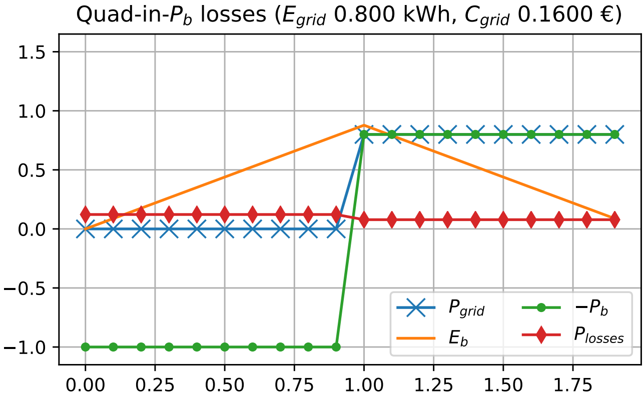
Case 2: PWL-in-P model



Losses reduce the gain, but no other effect on the profile

Production shifting experiment

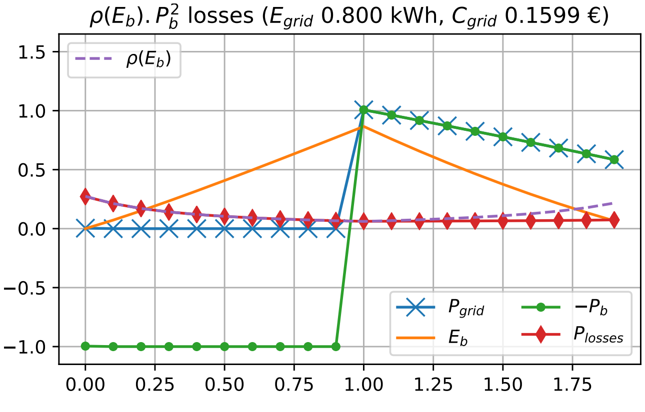
Case 3: Quadratic-in-P model



Quadratic losses *smooth out* the charge/discharge power

Production shifting experiment

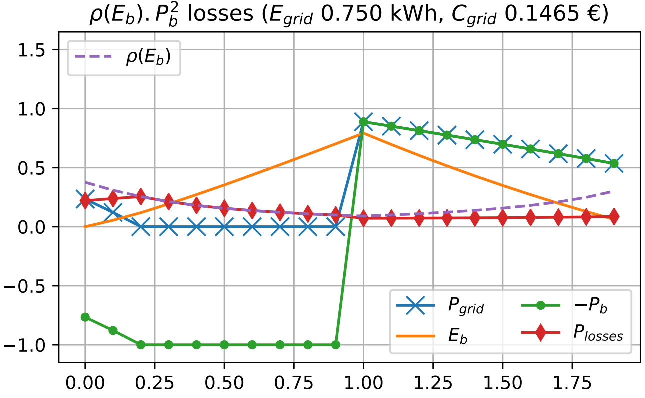
Case 4a: P^2/E model (supercapacitor)



Discharge power is reduced at low *SoE*

Production shifting experiment

Case 4b: P^2/E model (supercapacitor), 75% round-trip efficiency



Charging reduced at low *SoE*, energy given away to the grid

Tools

Optimization results obtained by:

- describing optimization problems in `Julia`
- using the `JuMP` package `https://jump.dev/`
- optimization solvers: Ipopt (NLP) and ECOS (LP, SOCP)

Results are open source

The complete code is available in a Jupyter notebook
`https://github.com/pierre-haessig/convex-storage-loss`

(including one example where the relaxation fails: negative energy price)

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Contribution:

- Unified description of storage loss relaxation (linear & nonlinear cases)
- One loss model to bind them all

Significance

Better loss models unleash more realistic storage trajectories

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Future work:

- Characterize the worst-case amount of artificially wasted energy, when the relaxation fails (negative energy price)

More details in a submitted conf. paper, soon available on HAL.